A predictive 3-3-1 model with A_4 flavor symmetry.

A. E. Cárcamo Hernández

Universidad Técnica Federico Santa María
and Centro Científico-Tecnológico de Valparaíso
Casilla 110-V, Valparaíso, Chile.

R. Martinez

Universidad Nacional de Colombia, Departamento de Física, Ciudad Universitaria, Bogotá D.C., Colombia.

Abstract

We propose a predictive model based on the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ gauge group supplemented by the $A_4 \otimes Z_3 \otimes Z_4 \otimes Z_6 \otimes Z_{16}$ discrete group, which successfully describes the SM fermion mass and mixing pattern. The small active neutrino masses are generated via inverse seesaw mechanism with three very light Majorana neutrinos. The observed charged fermion mass hierarchy and quark mixing pattern are originated from the breaking of the $Z_4 \otimes Z_6 \otimes Z_{16}$ discrete group at very high scale. The obtained values for the physical observables for both quark and lepton sectors are in excellent agreement with the experimental data. The model predicts a vanishing leptonic Dirac CP violating phase as well as an effective Majorana neutrino mass parameter of neutrinoless double beta decay, with values $m_{\beta\beta} = 2$ and 48 meV for the normal and the inverted neutrino mass hierarchies, respectively.

Keywords: Fermion masses and mixings, Discrete flavor symmetries, 3-3-1 models, Models beyond the Standard Model.

1. Introduction

Despite the great success of the Standard Model (SM), recently confirmed by the discovery of the 126 GeV Higgs boson by LHC experiments [1, 2, 3, 4], there are many aspects not yet explained such as the origin of the fermion mass and mixing hierarchy as well as the mechanism responsible for stabilizing the electroweak scale [5, 6]. This discovery of the Higgs scalar field allows to consider extensions of the SM with additional scalar fields that can be useful to explain the existence of Dark Matter [7].

The Standard Model is a theory with many phenomenological achievements. However in the Yukawa sector of the SM there are many parameters related with the fermion masses with no clear dynamical origin. Because of this reason, it is important to study realistic models that allow to set up relations among all these parameters of the Yukawa sector. Discrete flavor symmetries allow to establish ansatz that explain the flavor problem, for recent reviews see Refs. [8, 9, 10]. These discrete flavor symmetries may be crucial in building models of fermion mixing that address the flavor problem. Non abelian discrete flavor symmetries arise in string theories due to the discrete

features of the fixed points of the orbifolds [11]. For instance, the discrete D_4 group is originated in the S^1/Z_2 orbifold [11].

Besides that, another of the greatest mysteries in particle physics is the existence of three fermion families at low energies. The quark mixing angles are small whereas the leptonic mixing angles are large. Models based on the gauge symmetry $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ have the feature of being vectorlike with three families of fermions and are therefore anomaly free [12, 13, 14, 15, 16]. When the electric charge is defined in the linear combination of the $SU(3)_L$ generators T_3 and T_8, it is a free parameter, independent of the anomalies (β). The choice of this parameter defines the charge of the exotic particles. Choosing $\beta = -\frac{1}{\sqrt{3}}$, the third component of the weak lepton triplet is a neutral field ν_R^C, which allows to build the Dirac matrix with the usual field ν_L of the weak doublet. If one introduces a sterile neutrino N_R in the model, then it is possible to generate light neutrino masses via inverse seesaw mechanism. The 3-3-1 models with $\beta = -\frac{1}{\sqrt{3}}$ have the advantange of providing an alternative framework to generate neutrino masses, where the neutrino spectrum includes the light active sub-eV scale neutrinos as well as sterile neutrinos which could be dark matter candidates if they are light enough or candidates for detection at the LHC, if their masses are at the TeV scale. This interesting feature makes the 3-3-1 models very interesting, since if the TeV scale sterile neutrinos are found at the LHC, these models can be very strong candidates for unraveling the mechanism responsible for electroweak symmetry breaking. Furthermore, the 3-3-1 models can provide an explanation for the 750 GeV diphoton excess recently reported by ATLAS and CMS [17] as well as for the 2 TeV diboson excess found by ATLAS [18].

Neutrino oscillation experiments [6, 19, 20, 21, 22, 23] indicate that there are at least two massive active neutrinos and at most one massless active neutrino. In the mass eigenstates, it is necessary for the solar neutrinos oscillations that $\delta m^2_{sun} = m^2_{21} = m^2_2 - m^2_1$ where $m^2_2 - m^2_1 > 0$. For the atmospheric neutrinos oscillations it is required that $\delta m^2_{atm} = m^2_{31} = m^2_3 - m^2_1$ where the difference can be possitive (normal hierarchy) or negative (inverted hierarchy). Neutrino oscillations do not give information neither on the absolute value of the neutrino mass nor on the Majorana or Dirac nature of the neutrino. However there are neutrino mass bounds arising from cosmology [24], tritium beta decay [25] and double beta decay [26, 27, 28, 29, 30, 31, 32, 34, 33].

The neutrino masses and mixings are known from neutrino oscillations, which depend on the squared neutrino mass diferences and not on the absolute value of the neutrino masses. The global fits of the available data from the Daya Bay [19], T2K [20], MINOS [21], Double CHOOZ [22] and RENO [23] neutrino oscillation experiments, constrain the neutrino mass squared splittings and mixing parameters [35]. The current neutrino data on neutrino mixing parameters can be very well accommodated in the approximated tribimaximal mixing matrix,

$$U_{TBM} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix}, \tag{1}$$

which is consistent with two large mixing angles and one very small one of order zero. Specifically, the mixing angles predicted by the tribimaximal mixing matrix satisfy $\left(\sin^2\theta_{12}\right)_{TBM} = \frac{1}{3}$, $\left(\sin^2\theta_{23}\right)_{TBM} = \frac{1}{2}$, and $\left(\sin^2\theta_{13}\right)_{TBM} = 0$. However, the 3-3-1 model is not able to generate the tribimaximal matrix structure. Because of this reason, discrete symmetry groups [36, 37, 38, 41, 39, 40, 42, 43, 44, 45] that act on the fermion families are imposed with the aim to generate ansatz

that reproduce these matrices. One of the most promising discrete flavor groups is A_4, since it is the smallest symmetry with one three-dimensional and three distinct one-dimensional irreducible representations, where the three families of fermions can be accommodated rather naturally. Another approach to describe the fermion mass and mixing pattern consists in postulating particular mass matrix textures (see Ref [46] for some works considering textures). Besides that, models with Multi-Higgs sectors, Grand Unification, Extradimensions and Superstrings as well as with horizontal symmetries have been proposed in the literature [8, 47, 48, 49, 50] to explain the observed pattern of fermion masses and mixings.

In this paper we propose a version of the $SU(3)_C \times SU(3)_L \times U(1)_X$ model with an additional flavor symmetry group $A_4 \otimes Z_3 \otimes Z_4 \otimes Z_6 \otimes Z_{16}$ and an extended scalar sector needed in order to reproduce the specific patterns of mass matrices in the fermion sector that successfully account for fermion masses and mixings. The particular role of each additional scalar field and the corresponding particle assignments under the symmetry group of the model are explained in detail in Sec. 2. The model we are building with the aforementioned discrete symmetries, preserves the content of particles of the minimal 3-3-1 model, but we add additional very heavy scalar fields with quantum numbers that allow to build Yukawa terms invariant under the local and discrete groups. This generates the predictive and viable textures that explain the 18 physical observables in the quark and lepton sectors, i.e., the 9 charged fermion masses, 2 neutrino mass squared splittings, 3 lepton mixing parameters, 3 quark mixing angles and 1 CP violating phase of the CKM quark mixing matrix. Our model successfully describes the prevailing pattern of the SM fermion masses and mixing.

The content of this paper is organized as follows. In Sec. 2 we outline the proposed model. In Sec. 3 we discusss lepton masses and mixings and show our corresponding results. Our results for the masses and mixings in the quark sector followed by a numerical analysis are presented in Sec. 4. Finally in Sec. 5, we state our conclusions. In Appendix A we present a brief description of the A_4 group.

2. The model

We extend the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ group of the minimal 3-3-1 model by adding an extra flavor symmetry group $A_4 \otimes Z_3 \otimes Z_4 \otimes Z_6 \otimes Z_{16}$, in such a way that the full symmetry $\mathcal{G}$ experiences a three-step spontaneous breaking, as follows:

$$\begin{aligned}
&\mathcal{G} = SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes A_4 \otimes Z_3 \otimes Z_4 \otimes Z_6 \otimes Z_{16} \\
&\xrightarrow{\Lambda_{int}} SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes Z_3 \xrightarrow{v_\chi} SU(3)_C \otimes SU(2)_L \otimes U(1)_Y \\
&\xrightarrow{v_\eta, v_\rho} SU(3)_C \otimes U(1)_Q\,,
\end{aligned} \tag{2}$$

where the different symmetry breaking scales satisfy the following hierarchy $v_\eta, v_\rho \ll v_\chi \ll \Lambda_{int}$.

We define the electric charge in our 3-3-1 model in terms of the $SU(3)$ generators and the identity, as follows:

$$Q = T_3 - \frac{1}{\sqrt{3}} T_8 + XI, \tag{3}$$

with $I = Diag(1,1,1)$, $T_3 = \frac{1}{2}Diag(1,-1,0)$ and $T_8 = (\frac{1}{2\sqrt{3}})Diag(1,1,-2)$.

The anomaly cancellation of $SU(3)_L$ requires that the two families of quarks be accommodated in 3^* irreducible representations (irreps). Besides that, the number of 3^* irreducible representations is six, as follows from the quark colors. We accommodate the other family of quarks into a 3 irreducible representation. Furthermore, we have six 3 irreps taking into account the three families of leptons. Thus, the $SU(3)_L$ representations are vector like and free of anomalies. Having anomaly free $U(1)_X$ representations requires that the quantum numbers for the fermion families be assigned in such a way that the combination of the $U(1)_X$ representations with other gauge sectors cancels anomalies. Consequently, to avoid anomalies, the fermions have to be accommodated into the following $(SU(3)_C, SU(3)_L, U(1)_X)$ left- and right-handed representations:

$$
\begin{gathered}
Q_L^{1,2} = \begin{pmatrix} D^{1,2} \\ -U^{1,2} \\ J^{1,2} \end{pmatrix}_L : (3, 3^*, 0), \qquad Q_L^3 = \begin{pmatrix} U^3 \\ D^3 \\ T \end{pmatrix}_L : (3, 3, 1/3), \qquad L_L^{1,2,3} = \begin{pmatrix} \nu^{1,2,3} \\ e^{1,2,3} \\ (\nu^{1,2,3})^c \end{pmatrix}_L : (1, 3, -1/3), \\
\begin{array}{ll}
D_R^{1,2} : (3, 1, -1/3), & U_R^3 : (3, 1, 2/3), \\
U_R^{1,2} : (3, 1, 2/3), & D_R^3 : (3, 1, -1/3), \\
J_R^{1,2} : (3, 1, -1/3), & T_R : (3, 1, 2/3),
\end{array} \\
\begin{array}{lll}
e_R : (1, 1, -1), & \mu_R : (1, 1, -1), & \tau_R : (1, 1, -1), \\
N_R^1 : (1, 1, 0), & N_R^2 : (1, 1, 0), & N_R^3 : (1, 1, 0),
\end{array}
\end{gathered} \tag{4}
$$

where U_L^i and D_L^i for $i = 1, 2, 3$ are three up- and down-type quark components in the flavor basis, while ν_L^i and e_L^i (e_L, μ_L, τ_L) are the neutral and charged lepton families. The right-handed fermions are assigned as $SU(3)_L$ singlets representations having $U(1)_X$ quantum numbers equal to their electric charges. Furthermore, the fermion spectrum of the model includes as heavy fermions: a single flavor quark T with electric charge $2/3$, two flavor quarks $J^{1,2}$ with charge $-1/3$, three neutral Majorana leptons $(\nu^{1,2,3})_L^c$ and three right-handed Majorana leptons $N_R^{1,2,3}$ (see Ref. [51] for a recent discussion about neutrino masses via double and inverse see-saw mechanism for a 3-3-1 model).

The 3-3-1 models extend the scalar sector of the SM into three 3's irreps of $SU(3)_L$, where one heavy triplet χ acquires a vaccuum expectation value (VEV) at the TeV scale, v_χ, breaking the $SU(3)_L \times U(1)_X$ symmetry down to the $SU(2)_L \times U(1)_Y$ electroweak group of the SM and then giving masses to the non SM fermions and gauge bosons; and two lighter triplet fields η and ρ that get VEVs v_η and v_ρ, respectively, at the electroweak scale thus generating the mass for the fermion and gauge sector of the SM. We enlarge the scalar sector of the minimal 3-3-1 model by introducing 14 $SU(3)_L$ scalar singlets, namely, ξ_j, ζ_j, S_j , φ, Δ, ϕ, τ and σ ($j = 1, 2, 3$).

The scalars of our model are accommodated into the following $[SU(3)_L, U(1)_X]$ representations:

$$
\begin{aligned}
\chi &= \begin{pmatrix} \chi_1^0 \\ \chi_2^- \\ \frac{1}{\sqrt{2}}(\upsilon_\chi + \xi_\chi \pm i\zeta_\chi) \end{pmatrix} : (3, -1/3), \qquad \xi_j : (1, 0), \qquad \tau : (1, 0), \qquad \varphi : (1, 0), \qquad j = 1, 2, 3, \\
\rho &= \begin{pmatrix} \rho_1^+ \\ \frac{1}{\sqrt{2}}(\upsilon_\rho + \xi_\rho \pm i\zeta_\rho) \\ \rho_3^+ \end{pmatrix} : (3, 2/3), \qquad \zeta_j : (1, 0), \qquad \phi : (1, 0), \qquad \Delta : (1, 0), \qquad j = 1, 2, 3, \\
\eta &= \begin{pmatrix} \frac{1}{\sqrt{2}}(\upsilon_\eta + \xi_\eta \pm i\zeta_\eta) \\ \eta_2^- \\ \eta_3^0 \end{pmatrix} : (3, -1/3), \qquad S_j : (1, 0), \qquad \sigma \sim (1, 0), \qquad j = 1, 2, 3.
\end{aligned} \tag{5}
$$

The scalar fields are grouped into triplet and singlet represention of A_4. The scalar fields of our model have the following assignments under $A_4\otimes Z_3\otimes Z_4\otimes Z_6\otimes Z_{16}$:

$$\begin{aligned}
\eta &\sim \left(\mathbf{1},e^{-\frac{2i\pi}{3}},1,1,1\right), \quad \rho\sim\left(\mathbf{1},e^{\frac{2i\pi}{3}},1,1,1\right), \quad \chi\sim(\mathbf{1},1,1,1,1),\\
\xi &\sim (\mathbf{3},1,1,1,-1), \quad \zeta\sim\left(\mathbf{3},1,1,1,e^{\frac{i\pi}{8}}\right), \quad S\sim\left(\mathbf{3},e^{-\frac{2i\pi}{3}},1,1,e^{\frac{i\pi}{8}}\right), \quad \sigma\sim\left(\mathbf{1},1,1,1,e^{-\frac{i\pi}{8}}\right),\\
\varphi &\sim \left(\mathbf{1},1,1,e^{-\frac{i\pi}{3}},1\right), \quad \Delta\sim\left(\mathbf{1},1,-1,e^{-\frac{i\pi}{3}},1\right), \quad \phi\sim(\mathbf{1}',1,i,1,1), \quad \tau\sim(\mathbf{1}'',1,i,1,1), \quad (6)
\end{aligned}$$

where the numbers in boldface are dimensions of the A_4 irreducible representations.
The leptons transform under $A_4\otimes Z_3\otimes Z_4\otimes Z_6\otimes Z_{16}$ as:

$$\begin{aligned}
L_L &\sim \left(\mathbf{3},e^{\frac{2i\pi}{3}},1,1,-1\right), \quad e_R\sim\left(\mathbf{1},1,1,1,e^{\frac{7i\pi}{8}}\right), \quad \mu_R\sim(\mathbf{1}',1,1,1,i),\\
\tau_R &\sim \left(\mathbf{1}'',1,1,1,e^{\frac{i\pi}{4}}\right), \quad N_R\sim\left(\mathbf{3},e^{\frac{2i\pi}{3}},1,1,-1\right). \quad (7)
\end{aligned}$$

Note that left handed leptons are unified into a A_4 triplet representation $\mathbf{3}$, whereas the right handed charged leptons are assigned into different A_4 singlets, i.e, $\mathbf{1}$, $\mathbf{1}'$ and $\mathbf{1}''$. Furthermore, the right handed Majorana neutrinos are unified into a A_4 triplet representation.
The $A_4\otimes Z_3\otimes Z_4\otimes Z_6\otimes Z_{16}$ assignments for the quark sector are:

$$\begin{aligned}
Q_L^1 &\sim \left(\mathbf{1},1,1,1,e^{-\frac{i\pi}{8}}\right), \quad Q_L^2\sim(\mathbf{1}',1,1,1,1), \quad Q_L^3\sim(\mathbf{1}'',1,1,1,1),\\
U_R^1 &\sim \left(\mathbf{1},e^{-\frac{2\pi i}{3}},1,1,e^{\frac{7i\pi}{8}}\right), \quad U_R^2\sim\left(\mathbf{1}',e^{-\frac{2\pi i}{3}},1,1,i\right), \quad U_R^3\sim\left(\mathbf{1}'',e^{-\frac{2\pi i}{3}},1,1,1\right),\\
D_R^1 &\sim \left(\mathbf{1},e^{\frac{2\pi i}{3}},1,-1,e^{\frac{i\pi}{8}}\right), \quad D_R^2\sim\left(\mathbf{1},e^{\frac{2\pi i}{3}},1,-1,1\right), \quad D_R^3\sim\left(\mathbf{1}'',e^{\frac{2\pi i}{3}},1,-1,1\right)\\
T_R &\sim (\mathbf{1}'',1,1,1,1), \quad J_R^1\sim(\mathbf{1}',1,1,1,1) \quad J_R^2\sim(\mathbf{1}'',1,1,1,1). \quad (8)
\end{aligned}$$

With the above particle content, the following relevant Yukawa terms for the quark and lepton sector arise:

$$\begin{aligned}
-\mathcal{L}_Y^{(Q)} &= y_{11}^{(U)}\overline{Q}_L^1\rho^*U_R^1\frac{\sigma^8}{\Lambda^8}+y_{22}^{(U)}\overline{Q}_L^2\rho^*U_R^2\frac{\sigma^4}{\Lambda^2}+y_{33}^{(U)}\overline{Q}_L^3\eta U_R^3\\
&+y_{11}^{(D)}\overline{Q}_L^1\eta^*D_R^1\frac{\tau\phi\Delta^3\sigma^2}{\Lambda^7}+y_{12}^{(D)}\overline{Q}_L^1\eta^*D_R^2\frac{\tau\phi\Delta^3\sigma}{\Lambda^6}+y_{13}^{(D)}\overline{Q}_L^1\eta^*D_R^3\frac{\phi^2\Delta^3\sigma}{\Lambda^6}\\
&+y_{21}^{(D)}\overline{Q}_L^2\eta^*D_R^1\frac{\tau^2\Delta^3\sigma}{\Lambda^6}+y_{22}^{(D)}\overline{Q}_L^2\eta^*D_R^2\frac{\tau^2\Delta^3}{\Lambda^5}+y_{23}^{(D)}\overline{Q}_L^2\eta^*D_R^3\frac{\phi^2\Delta^3}{\Lambda^5}\\
&+y_{31}^{(D)}\overline{Q}_L^3\rho D_R^1\frac{\phi^2\Delta^3\sigma}{\Lambda^6}+y_{32}^{(D)}\overline{Q}_L^3\rho D_R^2\frac{\phi^2\Delta^3}{\Lambda^5}+y_{33}^{(D)}\overline{Q}_L^3\rho D_R^3\frac{\varphi^3}{\Lambda^3}\\
&+y^{(T)}\overline{Q}_L^3\chi T_R+y_1^{(J)}\overline{Q}_L^1\chi^*J_R^1+y_2^{(J)}\overline{Q}_L^2\chi^*J_R^2 \quad (9)
\end{aligned}$$

$$\begin{aligned}
-\mathcal{L}_Y^{(L)} &= h_{\rho e}^{(L)}\left(\overline{L}_L\rho\xi\right)_{\mathbf{1}}e_R\frac{\sigma^7}{\Lambda^8}+h_{\rho\mu}^{(L)}\left(\overline{L}_L\rho\xi\right)_{\mathbf{1}''}\mu_R\frac{\sigma^4}{\Lambda^5}+h_{\rho\tau}^{(L)}\left(\overline{L}_L\rho\xi\right)_{\mathbf{1}'}\tau_R\frac{\sigma^2}{\Lambda^3}\\
&+ h_\chi^{(L)}\left(\overline{L}_L\chi N_R\right)_{\mathbf{1}}+\frac{1}{2}h_{1N}\left(\overline{N}_RN_R^C\right)_{\mathbf{1}}\frac{\left(\eta^\dagger\cdot\eta^*\right)+x\left(\rho^T\cdot\rho\right)}{\Lambda}+h_{2N}\left(\overline{N}_RN_R^C\right)_{\mathbf{3s}}\frac{S\sigma}{\Lambda}\\
&+ h_\rho\varepsilon_{abc}\left(\overline{L}_L^a\left(L_L^C\right)^b\right)_{\mathbf{3s}}(\rho^*)^c\frac{\zeta\sigma}{\Lambda^2}+H.c, \quad (10)
\end{aligned}$$

where the dimensionless couplings $y_{ii}^{(U)}$, $y_{ij}^{(D)}$ $(i, j = 1, 2, 3)$, $y^{(T)}$, $y_1^{(J)}$, $y_2^{(J)}$, $h_{\rho e}^{(L)}$, $h_{\rho\mu}^{(L)}$, $h_{\rho\tau}^{(L)}$, $h_{\chi}^{(L)}$, h_{1N}, x, h_{2N} and h_ρ are $O(1)$ parameters. Here we assumed that all Yukawa couplings are real, excepting $y_{13}^{(D)}$, $y_{31}^{(D)}$ and $h_{\rho\tau}^{(L)}$ which are assumed to be complex.

Although the flavor discrete groups in Eq. (2) look rather sofisticated, each discrete group factor plays its own role in generating predictive fermion textures that successfully account for the low energy fermion flavor data. To describe the pattern of fermion masses and mixing angles, one needs to postulate particular Yukawa textures. As we will see in the next sections, the predictive textures for the lepton and quark sectors will give rise to the experimentally observed deviation of the tribimaximal mixing pattern and to quark mixing emerging only from the down type quark sector, respectively. A candidate for generating specific Yukawa textures is the A_4 flavor symmetry, which needs to be supplemented by the $Z_3 \otimes Z_4 \otimes Z_6 \otimes Z_{16}$ discrete group. As we will see in the next sections, this predictive setup can successfully account for fermion masses and mixings. The inclusion of the A_4 discrete group reduces the number of parameters in the Yukawa and scalar sector of the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ model making it more predictive. We choose A_4 since it is the smallest discrete group with a three-dimensional irreducible representation and 3 distinct one-dimensional irreducible representations, which allows to naturally accommodate the three fermion families. We unify the left-handed leptons in the A_4 triplet representation and the right-handed leptons are assigned to A_4 singlets. Regarding the quark sector, we assign quarks into A_4 singlet representations. In what follows we describe the role of each discrete cyclic group factor introduced in our model. The Z_3 symmetry separates the A_4 scalar triplets participating in the Yukawa interactions for charged leptons from those ones participating in the neutrino Yukawa interactions. Besides that, the Z_3 symmetry avoids mixings between SM quarks and exotic quarks since the right handed exotic quarks are neutral under this symmetry whereas the right handed SM quarks have non trivial Z_3 charges. Thus the Z_3 symmetry decouples the SM quarks from the exotic quarks resulting in a reduction of quark sector model parameters. Furthermore, the Z_4 symmetry is also important for reducing the number of quark sector model parameters, since due to this symmetry, the $SU(3)_L$ scalar singlets A_4 nontrivial singlets only appear in the down type quark Yukawa terms. Consequently this Z_4 symmetry together with the A_4 assignments for quarks described in Eq. (8), results in a diagonal up type quark mass matrix, thus giving rise to a quark mixing only emerging from the down type quark sector. The Z_6 symmetry is crucial for explaining the hierarchy between the SM down and SM up type quarks without tuning the SM down type quark Yukawa couplings, since it is the smallest cyclic symmetry that allows $\frac{\varphi^3}{\Lambda^3}$ in the Yukawa term that generates the bottom quark mass, which is $\lambda^3 \frac{v}{\sqrt{2}}$ ($\lambda = 0.225$ is one of the Wolfenstein parameters) times a $O(1)$ parameter. The Z_{16} symmetry gives rise to the observed hierarchy among charged fermion masses and quark mixing angles. It is worth mentioning that the properties of the Z_N groups imply that the Z_{16} symmetry is the smallest cyclic symmetry that allows to build the Yukawa term $\overline{Q}_L^1 \rho^* U_R^1 \frac{\sigma^8}{\Lambda^8}$ of dimension twelve from a $\frac{\sigma^8}{\Lambda^8}$ insertion on the $\overline{Q}_L^1 \rho^* U_R^1$ operator, crucial to get the required λ^8 suppression (where $\lambda = 0.225$ is one of the Wolfenstein parameters) needed to naturally explain the smallness of the up quark mass. Regarding the charged lepton sector, let us note that the five dimensional Yukawa operators $\frac{1}{\Lambda}\left(\overline{L}_L \rho \xi\right)_{\mathbf{1}} e_R$, $\frac{1}{\Lambda}\left(\overline{L}_L \rho \xi\right)_{\mathbf{1}''} \mu_R$ and $\frac{1}{\Lambda}\left(\overline{L}_L \rho \xi\right)_{\mathbf{1}'} \tau_R$ are A_4 invariant but do not preserve the Z_{16} symmetry, as follows from the charges assignments given by Eqs. (6) and (7).

In what follows we comment about the possible VEVs patterns for the A_4 scalar triplets ξ, ζ and S. Here we assume a hierarchy between the VEVs of the A_4 scalar triplets ξ, ζ and S, i.e., $v_S << v_\zeta << v_\xi$, which implies that the mixing angles of these scalar triplets are very small

since they are suppressed by the ratios of their VEVs, which is a consequence of the method of recursive expansion proposed in Ref. [52]. Consequently, we can neglect the mixing between the A_4 scalar triplets ξ, ζ and S, and treat their corresponding scalar potentials independently. The relevant terms determining the VEV directions of any A_4 acalar triplet are:

$$\begin{aligned} V(\Sigma) &= -\mu_\Sigma^2 (\Sigma\Sigma^*)_{\mathbf{1}} + \kappa_{\Sigma,1} (\Sigma\Sigma^*)_{\mathbf{1}} (\Sigma\Sigma^*)_{\mathbf{1}} + \kappa_{\Sigma,2} (\Sigma\Sigma)_{\mathbf{1}} (\Sigma^*\Sigma^*)_{\mathbf{1}} + \kappa_{\Sigma,3} (\Sigma\Sigma^*)_{\mathbf{1}'} (\Sigma\Sigma^*)_{\mathbf{1}''} \\ &+ \kappa_{\Sigma,4} \left[(\Sigma\Sigma)_{\mathbf{1}'} (\Sigma^*\Sigma^*)_{\mathbf{1}''} + h.c\right] + \kappa_{\Sigma,5} \left[(\Sigma\Sigma)_{\mathbf{1}''} (\Sigma^*\Sigma^*)_{\mathbf{1}'} + h.c\right] \\ &+ \kappa_{\Sigma,6} (\Sigma\Sigma^*)_{\mathbf{3s}} (\Sigma\Sigma^*)_{\mathbf{3s}} + \kappa_{\Sigma,7} (\Sigma\Sigma)_{\mathbf{3s}} (\Sigma^*\Sigma^*)_{\mathbf{3s}} . \end{aligned} \tag{11}$$

where $\Sigma = \xi, \zeta, S$.

The part of the scalar potential for each A_4 scalar triplet has 8 free parameters: 1 bilinear and 7 quartic couplings. The minimization conditions of the scalar potential for a A_4 triplet yield the following relations:

$$\begin{aligned} \frac{\partial \langle V(\Sigma)\rangle}{\partial v_{\Sigma_1}} &= -2v_{\Sigma_1}\mu_\Sigma^2 + 4\kappa_{\Sigma,1}v_{\Sigma_1}\left(v_{\Sigma_1}^2 + v_{\Sigma_2}^2 + v_{\Sigma_3}^2\right) + 2\kappa_{\Sigma,3}v_{\Sigma_1}\left(2v_{\Sigma_1}^2 - v_{\Sigma_2}^2 - v_{\Sigma_3}^2\right) \\ &+ 4\kappa_{\Sigma,2}v_{\Sigma_1}\left[v_{\Sigma_1}^2 + v_{\Sigma_2}^2\cos(2\theta_{\Sigma_1} - 2\theta_{\Sigma_2}) + v_{\Sigma_3}^2\cos(2\theta_{\Sigma_1} - 2\theta_{\Sigma_3})\right] + 8\kappa_{\Sigma,7}v_{\Sigma_1}\left(v_{\Sigma_2}^2 + v_{\Sigma_3}^2\right) \\ &+ 4(\kappa_{\Sigma,4} + \kappa_{\Sigma,5})v_{\Sigma_1}\left[2v_{\Sigma_1}^2 - v_{\Sigma_2}^2\cos(2\theta_{\Sigma_1} - 2\theta_{\Sigma_2}) - v_{\Sigma_3}^2\cos(2\theta_{\Sigma_1} - 2\theta_{\Sigma_3})\right] \\ &+ 4\kappa_{\Sigma,6}v_{\Sigma_1}\left[v_{\Sigma_2}^2\{1 + \cos(2\theta_{\Sigma_1} - 2\theta_{\Sigma_2})\} + v_{\Sigma_3}^2\{1 + \cos(2\theta_{\Sigma_1} - 2\theta_{\Sigma_3})\}\right] \\ &= 0, \\ \frac{\partial \langle V(\Sigma)\rangle}{\partial v_{\Sigma_2}} &= -2v_{\Sigma_2}\mu_\Sigma^2 + 4\kappa_{\Sigma,1}v_{\Sigma_2}\left(v_{\Sigma_1}^2 + v_{\Sigma_2}^2 + v_{\Sigma_3}^2\right) + 2\kappa_{\Sigma,3}v_{\Sigma_2}\left(2v_{\Sigma_2}^2 - v_{\Sigma_1}^2 - v_{\Sigma_3}^2\right) \\ &+ 4\kappa_{\Sigma,2}v_{\Sigma_2}\left[v_{\Sigma_2}^2 + v_{\Sigma_1}^2\cos(2\theta_{\Sigma_2} - 2\theta_{\Sigma_1}) + v_{\Sigma_3}^2\cos(2\theta_{\Sigma_2} - 2\theta_{\Sigma_3})\right] + 8\kappa_{\Sigma,7}v_{\Sigma_2}\left(v_{\Sigma_1}^2 + v_{\Sigma_3}^2\right) \\ &+ 4(\kappa_{\Sigma,4} + \kappa_{\Sigma,5})v_{\Sigma_2}\left[2v_{\Sigma_2}^2 - v_{\Sigma_1}^2\cos(2\theta_{\Sigma_2} - 2\theta_{\Sigma_1}) - v_{\Sigma_3}^2\cos(2\theta_{\Sigma_2} - 2\theta_{\Sigma_3})\right] \\ &+ 4\kappa_{\Sigma,6}v_{\Sigma_2}\left[v_{\Sigma_1}^2\{1 + \cos(2\theta_{\Sigma_2} - 2\theta_{\Sigma_1})\} + v_{\Sigma_3}^2\{1 + \cos(2\theta_{\Sigma_2} - 2\theta_{\Sigma_3})\}\right] \\ &= 0, \\ \frac{\partial \langle V(\Sigma)\rangle}{\partial v_{\Sigma_3}} &= -2v_{\Sigma_3}\mu_\Sigma^2 + 4\kappa_{\Sigma,1}v_{\Sigma_3}\left(v_{\Sigma_1}^2 + v_{\Sigma_2}^2 + v_{\Sigma_3}^2\right) + 2\kappa_{\Sigma,3}v_{\Sigma_3}\left(2v_{\Sigma_3}^2 - v_{\Sigma_1}^2 - v_{\Sigma_2}^2\right) \\ &+ 4\kappa_{\Sigma,2}v_{\Sigma_3}\left[v_{\Sigma_2}^2 + v_{\Sigma_1}^2\cos(2\theta_{\Sigma_1} - 2\theta_{\Sigma_2}) + v_{\Sigma_2}^2\cos(2\theta_{\Sigma_3} - 2\theta_{\Sigma_2})\right] + 8\kappa_{\Sigma,7}v_{\Sigma_3}\left(v_{\Sigma_1}^2 + v_{\Sigma_2}^2\right) \\ &+ 4(\kappa_{\Sigma,4} + \kappa_{\Sigma,5})v_{\Sigma_3}\left[2v_{\Sigma_3}^2 - v_{\Sigma_1}^2\cos(2\theta_{\Sigma_1} - 2\theta_{\Sigma_2}) - v_{\Sigma_2}^2\cos(2\theta_{\Sigma_3} - 2\theta_{\Sigma_2})\right] \\ &+ 4\kappa_{\Sigma,6}v_{\Sigma_3}\left[v_{\Sigma_1}^2\{1 + \cos(2\theta_{\Sigma_1} - 2\theta_{\Sigma_2})\} + v_{\Sigma_2}^2\{1 + \cos(2\theta_{\Sigma_3} - 2\theta_{\Sigma_2})\}\right] \\ &= 0. \end{aligned} \tag{12}$$

where $\langle\Sigma\rangle = \left(v_{\Sigma_1}e^{i\theta_{\Sigma_1}}, v_{\Sigma_2}e^{i\theta_{\Sigma_2}}, v_{\Sigma_3}e^{i\theta_{\Sigma_3}}\right)$. Here in order to simplify the analysis, we restrict to the simplest case of zero phases in the VEV patterns of the A_4 triplet scalars, i.e., $\theta_{\Sigma_1} = \theta_{\Sigma_2} = \theta_{\Sigma_3} = 0$. Then, from the scalar potential minimization equations given by Eq. (12), the following relations are obtained:

$$\begin{aligned} \left[3\kappa_{\Sigma,3} - 4(\kappa_{\Sigma,6} + \kappa_{\Sigma,7}) + 6(\kappa_{\Sigma,4} + \kappa_{\Sigma,5})\right]\left(v_{\Sigma_1}^2 - v_{\Sigma_2}^2\right) &= 0, \\ \left[3\kappa_{\Sigma,3} - 4(\kappa_{\Sigma,6} + \kappa_{\Sigma,7}) + 6(\kappa_{\Sigma,4} + \kappa_{\Sigma,5})\right]\left(v_{\Sigma_1}^2 - v_{\Sigma_3}^2\right) &= 0, \\ \left[3\kappa_{\Sigma,3} - 4(\kappa_{\Sigma,6} + \kappa_{\Sigma,7}) + 6(\kappa_{\Sigma,4} + \kappa_{\Sigma,5})\right]\left(v_{\Sigma_2}^2 - v_{\Sigma_3}^2\right) &= 0. \end{aligned} \tag{13}$$

From the relations given by Eq. (13) and setting $\kappa_{\zeta,3} = \frac{4}{3}\left(\kappa_{\zeta,6} + \kappa_{\zeta,7}\right) - 2\left(\kappa_{\zeta,4} + \kappa_{\zeta,5}\right)$, we obtain the following VEV pattern:

$$\langle \xi \rangle = \frac{v_\xi}{\sqrt{3}}(1,1,1), \qquad \langle \zeta \rangle = \frac{v_\zeta}{\sqrt{2}}(1,0,1), \qquad \langle S \rangle = \frac{v_S}{\sqrt{3}}(1,1,-1). \tag{14}$$

In the case of ξ, this is a vacuum configuration preserving a Z_3 subgroup of A_4, which has been extensively studied in many A_4 flavor models (for recent reviews see Refs. [8, 9, 10]). The VEV pattern for the A_4 triplet scalar ζ is similar to the one previously studied in an A_4 and T_7 flavor $SU(5)$ GUT models [38, 44] and in a 6HDM with A_4 flavor symmetry [37]. As we will see in the next section, the VEV patterns for the A_4 triplets ξ, ζ and S given in Eq. 14) are crucial to get a predictive model that successfully reproduces the experimental values of the physical observables in the lepton sector.

Furthermore we assume that these $SU(3)_L$ scalar singlets get VEVs at a scale Λ_{int} much larger than v_χ (which is of the order of the TeV scale), with the exception of S_j ($j = 1,2,3$), which get VEVs much smaller than the electroweak symmetry breaking scale $v = 246$ GeV. The VEVs of the ξ_j ($j = 1,2,3$), φ, Δ, ϕ, τ and σ scalar singlets break the $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes A_4 \otimes Z_3 \otimes Z_4 \otimes Z_6 \otimes Z_{16}$ symmetry down to $SU(3)_C \otimes SU(3)_L \otimes U(1)_X \otimes Z_3$ at the scale Λ_{int}.
From the expressions given above, and using the vacuum configuration for the A_4 scalar triplets given in Eq. (14), we find the following relations:

$$\begin{aligned}
\mu_\xi^2 &= \frac{2}{3}\left[3\left(\kappa_{\xi,1} + \kappa_{\xi,2}\right) + 4\left(\kappa_{\xi,6} + \kappa_{\xi,7}\right)\right] v_\xi^2, \\
\mu_\zeta^2 &= \frac{2}{3}\left[3\left(\kappa_{\zeta,1} + \kappa_{\zeta,2}\right) + 4\left(\kappa_{\zeta,6} + \kappa_{\zeta,7}\right)\right] v_\zeta^2, \\
\mu_S^2 &= \frac{2}{3}\left[3\left(\kappa_{S,1} + \kappa_{S,2}\right) + 4\left(\kappa_{S,6} + \kappa_{S,7}\right)\right] v_S^2.
\end{aligned} \tag{15}$$

These results show that the VEV directions for the three A_4 triplets, i.e., ξ, ζ and S scalars in Eq. (14), are consistent with a global minimum of the scalar potential (11) of our model for a large region of parameter space.

Besides that, as the hierarchy among charged fermion masses and quark mixing angles emerges from the breaking of the $Z_4 \otimes Z_6 \otimes Z_{16}$ discrete group, we set the VEVs of the $SU(3)_L$ singlet scalar fields ξ, φ, Δ, ϕ, τ and σ, with respect to the Wolfenstein parameter $\lambda = 0.225$ and the model cutoff Λ, as follows:

$$v_\varphi \sim v_\tau \sim v_\phi \sim v_\Delta \sim v_\xi \sim v_\sigma \sim \Lambda_{int} = \lambda\Lambda. \tag{16}$$

Furthermore, we assume that the A_4 scalar triplets participating in the neutrino Yukawa interactions have VEVs much smaller than the electroweak symmetry breaking scale. Besides that, as previously mentioned, we assume a hierarchy among the VEVs of the two A_4 scalar triplets participating in the neutrino Yukawa terms. Consequently, as we will see in the next section, the Majorana neutrinos acquire very small masses and thus an inverse seesaw mechanism for the generation of light active neutrino masses, takes place. Therefore, we have the following hierarchy among the VEVs of the scalar fields in our model:

$$v_S << v_\zeta << v_\rho \sim v_\eta \sim v << v_\chi << \Lambda_{int}. \tag{17}$$

In what follows, we briefly comment about the low energy scalar sector of our model. The renormalizable low energy scalar potential of the model is given by:

$$
\begin{aligned}
V_H \quad = \quad & \mu_\chi^2(\chi^\dagger\chi) + \mu_\eta^2(\eta^\dagger\eta) + \mu_\rho^2(\rho^\dagger\rho) + f\left(\chi_i\eta_j\rho_k\varepsilon^{ijk} + H.c.\right) + \lambda_1(\chi^\dagger\chi)(\chi^\dagger\chi) \\
& +\lambda_2(\rho^\dagger\rho)(\rho^\dagger\rho) + \lambda_3(\eta^\dagger\eta)(\eta^\dagger\eta) + \lambda_4(\chi^\dagger\chi)(\rho^\dagger\rho) + \lambda_5(\chi^\dagger\chi)(\eta^\dagger\eta) \\
& +\lambda_6(\rho^\dagger\rho)(\eta^\dagger\eta) + \lambda_7(\chi^\dagger\eta)(\eta^\dagger\chi) + \lambda_8(\chi^\dagger\rho)(\rho^\dagger\chi) + \lambda_9(\rho^\dagger\eta)(\eta^\dagger\rho).
\end{aligned} \tag{18}
$$

After the symmetry breaking takes place, it is found that the scalar mass eigenstates are related with the weak scalar states by: [14, 15]:

$$
\begin{pmatrix} G_1^\pm \\ H_1^\pm \end{pmatrix} = R_{\beta_T}\begin{pmatrix} \rho_1^\pm \\ \eta_2^\pm \end{pmatrix} \quad , \qquad \begin{pmatrix} G_1^0 \\ A_1^0 \end{pmatrix} = R_{\beta_T}\begin{pmatrix} \zeta_\rho \\ \zeta_\eta \end{pmatrix}, \quad \begin{pmatrix} H_1^0 \\ h^0 \end{pmatrix} = R_{\alpha_T}\begin{pmatrix} \xi_\rho \\ \xi_\eta \end{pmatrix}, \tag{19}
$$

$$
\begin{pmatrix} G_2^0 \\ H_2^0 \end{pmatrix} = R\begin{pmatrix} \chi_1^0 \\ \eta_3^0 \end{pmatrix} \quad , \qquad \begin{pmatrix} G_3^0 \\ H_3^0 \end{pmatrix} = R\begin{pmatrix} \zeta_\chi \\ \xi_\chi \end{pmatrix}, \quad \begin{pmatrix} G_2^\pm \\ H_2^\pm \end{pmatrix} = R\begin{pmatrix} \chi_2^\pm \\ \rho_3^\pm \end{pmatrix}, \tag{20}
$$

with

$$
R_{\alpha_T(\beta_T)} = \begin{pmatrix} \cos\alpha_T(\beta_T) & \sin\alpha_T(\beta_T) \\ -\sin\alpha_T(\beta_T) & \cos\alpha_T(\beta_T) \end{pmatrix}, \qquad\qquad R = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}, \tag{21}
$$

where $\tan\beta_T = v_\eta/v_\rho$, and $\tan 2\alpha_T = M_1/(M_2 - M_3)$ with:

$$
\begin{aligned}
M_1 &= 4\lambda_6 v_\eta v_\rho + 2\sqrt{2} f v_\chi, \\
M_2 &= 4\lambda_2 v_\rho^2 - \sqrt{2} f v_\chi \tan\beta_T, \\
M_3 &= 4\lambda_3 v_\eta^2 - \sqrt{2} f v_\chi / \tan\beta_T.
\end{aligned} \tag{22}
$$

It is noteworthy to mention that the our model has the following scalar states at low energies: 4 massive charged Higgs ($H_1^\pm$, $H_2^\pm$), one CP-odd Higgs (A_1^0), 3 neutral CP-even Higgs (h^0, H_1^0, H_3^0) and 2 neutral Higgs ($H_2^0, \overline{H}_2^0$) bosons. We identify the scalar h^0 with the SM-like 126 GeV Higgs boson discovered at the LHC. Let us note that the neutral Goldstone bosons G_1^0, G_3^0, G_2^0, $\overline{G}_2^0$ correspond to the longitudinal components of the Z, Z', K^0 and $\overline{K}^0$ gauge bosons, respectively. Besides that, the charged Goldstone bosons $G_1^\pm$ and $G_2^\pm$ are associated to the longitudinal components of the $W^\pm$ and $K^\pm$ gauge bosons, respectively [12, 15].

3. Lepton masses and mixings

From Eq. (10) and taking into account that the VEV pattern of the A_4 triplet, $SU(3)_L$ singlet scalar field ξ satisfies Eq. (14) with the nonvanishing VEVs of the $SU(3)_L$ singlet scalars ξ and σ, set to be equal to $\lambda\Lambda$ (being Λ the cutoff of our model) as indicated by Eq. (16), we find that the charged lepton mass matrix is given by:

$$M_l = R_{lL}^{\dagger} P_l diag\left(m_e, m_\mu, m_\tau\right), \quad R_{lL} = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}, \quad \omega = e^{\frac{2\pi i}{3}}, \quad P_l = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\alpha} \end{pmatrix}, \tag{23}$$

being α the complex phase of $h_{\rho\tau}^{(L)}$, and the charged lepton masses are:

$$m_e = a_1^{(l)} \lambda^8 \frac{v}{\sqrt{2}}, \qquad m_\mu = a_2^{(l)} \lambda^5 \frac{v}{\sqrt{2}}, \qquad m_\tau = a_3^{(l)} \lambda^3 \frac{v}{\sqrt{2}}. \tag{24}$$

where $\lambda = 0.225$ is one of the Wolfenstein parameters, $v = 246$ GeV the scale of electroweak symmetry breaking and $a_i^{(l)}$ ($i = 1, 2, 3$) are $O(1)$ parameters. Let us note that the charged lepton masses are linked with the scale of electroweak symmetry breaking through their power dependence on the Wolfenstein parameter $\lambda = 0.225$, with $O(1)$ coefficients. Furthermore, from the lepton Yukawa terms given in Eq. (10) it is easy to see that our model does not feature flavor changing leptonic neutral Higgs decays. Consequently, our model cannot explain the recently reported anomaly in the $h \to \mu\tau$ decay, implying that a measurement of its branching fraction will be decisive for its exclusion.

Regarding the neutrino sector, we can write the neutrino mass terms as:

$$-\mathcal{L}_{mass}^{(\nu)} = \frac{1}{2} \begin{pmatrix} \overline{\nu_L^C} & \overline{\nu_R} & \overline{N_R} \end{pmatrix} M_\nu \begin{pmatrix} \nu_L \\ \nu_R^C \\ N_R^C \end{pmatrix} + H.c, \tag{25}$$

where the neutrino mass matrix is constrained from the A_4 flavor symmetry and has the following form:

$$M_\nu = \begin{pmatrix} 0_{3\times3} & M_D & 0_{3\times3} \\ M_D^T & 0_{3\times3} & M_\chi \\ 0_{3\times3} & M_\chi^T & M_R \end{pmatrix}, \tag{26}$$

and the submatrices are given by:

$$M_D = \frac{h_\rho v_\rho v_\zeta v_\sigma}{2\Lambda^2} \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix}, \quad M_R = \begin{pmatrix} h_{1N} \frac{v_\eta^2 + x v_\rho^2}{\Lambda^2} v_\sigma & -h_{2N} \frac{v_S v_\sigma}{\sqrt{3}\Lambda} & h_{2N} \frac{v_S v_\sigma}{\sqrt{3}\Lambda} \\ -h_{2N} \frac{v_S v_\sigma}{\sqrt{3}\Lambda} & h_{1N} \frac{v_\eta^2 + x v_\rho^2}{\Lambda^2} v_\sigma & h_{2N} \frac{v_S v_\sigma}{\sqrt{3}\Lambda} \\ h_{2N} \frac{v_S v_\sigma}{\sqrt{3}\Lambda} & h_{2N} \frac{v_S v_\sigma}{\sqrt{3}\Lambda} & h_{1N} \frac{v_\eta^2 + x v_\rho^2}{\Lambda^2} v_\sigma \end{pmatrix},$$

$$M_\chi = h_\chi^{(L)} \frac{v_\chi}{\sqrt{2}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}. \tag{27}$$

As previously mentioned, we assume in our model that the $SU(3)_L$ scalar singlet, A_4 triplet S interacting with the right handed Majorana neutrinos gets a very small vacuum expectation value,

much smaller than the electroweak symmetry breaking scale, which results in very small masses for these Majorana neutrinos. Consequently, this setup can generate small active neutrino masses through an inverse seesaw mechanism.

As shown in detail in Ref. [51], the full rotation matrix is approximately given by:

$$\mathbb{U} = \begin{pmatrix} R_\nu & B_3 U_\chi & B_2 U_R \\ -\frac{(B_2^\dagger + B_3^\dagger)}{\sqrt{2}} R_\nu & \frac{(1-S)}{\sqrt{2}} U_\chi & \frac{(1+S)}{\sqrt{2}} U_R \\ -\frac{(B_2^\dagger - B_3^\dagger)}{\sqrt{2}} R_\nu & \frac{(-1-S)}{\sqrt{2}} U_\chi & \frac{(1-S)}{\sqrt{2}} U_R \end{pmatrix}, \tag{28}$$

where

$$S = -\frac{1}{2\sqrt{2} h_\chi^{(L)} v_\chi} M_R, \qquad B_2 \simeq B_3 \simeq \frac{1}{h_\chi^{(L)} v_\chi} M_D^*, \tag{29}$$

and the physical neutrino mass matrices are:

$$M_\nu^{(1)} = M_D \left(M_\chi^T\right)^{-1} M_R M_\chi^{-1} M_D^T, \tag{30}$$

$$M_\nu^{(2)} = -M_\chi^T + \frac{1}{2} M_R, \qquad M_\nu^{(3)} = M_\chi^T + \frac{1}{2} M_R, \tag{31}$$

where $M_\nu^{(1)}$ corresponds to the active neutrino mass matrix whereas $M_\nu^{(2)}$ and $M_\nu^{(3)}$ are the exotic Dirac neutrino mass matrices. Note that the physical eigenstates include three active neutrinos and six exotic neutrinos. The exotic neutrinos are pseudo-Dirac, with masses $\sim \pm M_\chi^T$ and a small splitting M_R. Furthermore, R_ν, U_R and U_χ are the rotation matrices which diagonalize $M_\nu^{(1)}$, $M_\nu^{(2)}$ and $M_\nu^{(3)}$, respectively [51].

From Eq. (30) it follows that the light active neutrino mass matrix is given by:

$$\begin{aligned} M_\nu^{(1)} &= -\frac{h_\rho^2 v_\rho^2 v_\zeta^2 v_\sigma}{2 h_\chi^{(L)} v_\chi^2 \Lambda^3} \begin{pmatrix} h_{1N} \frac{v_\chi^2}{\Lambda} & 0 & h_{1N} \frac{v_\chi^2}{\Lambda} \\ 0 & h_{1N} \frac{v_\chi^2}{\Lambda} + \frac{2 h_{2N}}{\sqrt{3}} v_S + h_{1N} \frac{v_\chi^2}{\Lambda} & 0 \\ h_{1N} \frac{v_\chi^2}{\Lambda} & 0 & h_{1N} \frac{v_\chi^2}{\Lambda} \end{pmatrix} = \begin{pmatrix} A & 0 & A \\ 0 & B & 0 \\ A & 0 & A \end{pmatrix}, \\ A &= -\frac{h_{1N} h_\rho^2 v_\rho^2 v_\zeta^2 v_\chi^2 v_\sigma}{2 h_\chi^{(L)} v_\chi^2 \Lambda^4}, \qquad B = -\frac{h_\rho^2 v_\rho^2 v_\zeta^2 v_\sigma}{2 h_\chi^{(L)} v_\chi^2 \Lambda^3} \left(h_{1N} \frac{v_\chi^2}{\Lambda} + \frac{2 h_{2N}}{\sqrt{3}} v_S + h_{1N} \frac{v_\chi^2}{\Lambda} \right). \end{aligned} \tag{32}$$

The neutrino mass matrix given in Eq. (32) only depends on two effective parameters: A and B. These effective parameters include the dependence on the various model parameters. It is noteworthy that A and B are suppressed by inverse powers of the high energy cutoff Λ of our model.

The light active neutrino mass matrix $M_\nu^{(1)}$ is diagonalized by a unitary rotation matrix R_ν, according to:

$$R_\nu^T M_\nu^{(1)} R_\nu = \begin{pmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{pmatrix}, \quad \text{with} \quad R_\nu = \begin{pmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{pmatrix}, \quad \theta = \pm\frac{\pi}{4}, \tag{33}$$

where the upper sign corresponds to normal ($\theta = +\pi/4$) and the lower one to inverted ($\theta = -\pi/4$) hierarchy, respectively. The light active neutrino masses for the normal (NH) and inverted (IH)

mass hierarchies are given by:

$$\text{NH} \quad : \quad \theta = +\frac{\pi}{4} : \qquad m_{\nu_1} = 0, \qquad m_{\nu_2} = B, \qquad m_{\nu_3} = 2A, \tag{34}$$

$$\text{IH} \quad : \quad \theta = -\frac{\pi}{4} : \qquad m_{\nu_1} = 2A, \qquad m_{\nu_2} = B, \qquad m_{\nu_3} = 0. \tag{35}$$

We also find that the PMNS leptonic mixing matrix is given by:

$$U = R_{lL}^{\dagger} P_l R_\nu \simeq \begin{pmatrix} \frac{\cos\theta}{\sqrt{3}} - \frac{\sin\theta}{\sqrt{3}} e^{i\alpha} & \frac{1}{\sqrt{3}} & \frac{\cos\theta}{\sqrt{3}} e^{i\alpha} + \frac{\sin\theta}{\sqrt{3}} \\ \frac{\cos\theta}{\sqrt{3}} - \frac{\sin\theta}{\sqrt{3}} e^{i\alpha + \frac{2i\pi}{3}} & \frac{1}{\sqrt{3}} e^{-\frac{2i\pi}{3}} & \frac{\cos\theta}{\sqrt{3}} e^{i\alpha + \frac{2i\pi}{3}} + \frac{\sin\theta}{\sqrt{3}} \\ \frac{\cos\theta}{\sqrt{3}} - \frac{\sin\theta}{\sqrt{3}} e^{i\alpha - \frac{2i\pi}{3}} & \frac{1}{\sqrt{3}} e^{\frac{2i\pi}{3}} & \frac{\cos\theta}{\sqrt{3}} e^{i\alpha - \frac{2i\pi}{3}} + \frac{\sin\theta}{\sqrt{3}} \end{pmatrix}. \tag{36}$$

It is worth commenting that the Pontecorvo-Maki-Nakagawa-Sakata (PMNS) mixing matrix depends only on the parameter α, while the neutrino mass squared splittings are controlled by parameters A and B.

The standard parametrization of the leptonic mixing matrix implies that the lepton mixing angles satisfy [6]:

$$\sin^2\theta_{12} = \frac{|U_{e2}|^2}{1 - |U_{e3}|^2} = \frac{1}{2 - \cos\alpha}, \qquad \sin^2\theta_{13} = |U_{e3}|^2 = \frac{1}{3}(1 + \cos\alpha), \tag{37}$$

$$\sin^2\theta_{23} = \frac{\left|U_{\mu 3}\right|^2}{1 - |U_{e3}|^2} = \frac{1}{2}\left(1 + \frac{\sqrt{3}\sin\alpha}{\cos\alpha - 2}\right).$$

The resulting PMNS matrix (36) reduces to the trimaximal mixing matrix (1) in the limit $\alpha = \pi$, for the inverted and normal hierarchies of the neutrino mass spectrum. Let us note that the lepton mixing angles are controlled by a single parameter (α), whereas the neutrino mass squared splittings only depend on the parameters A and B.

The Jarlskog invariant and the CP violating phase are [6]:

$$J = \mathrm{Im}\left(U_{e1} U_{\mu 2} U_{e2}^{*} U_{\mu 1}^{*}\right) = -\frac{1}{6\sqrt{3}} \cos 2\theta, \qquad \sin\delta = \frac{8J}{\cos\theta_{13} \sin 2\theta_{12} \sin 2\theta_{23} \sin 2\theta_{13}}. \tag{38}$$

Taking into account that $\theta = \pm\frac{\pi}{4}$, our model predicts $J = 0$ and $\delta = 0$, which results in a vanishing leptonic Dirac CP violating phase.

In what follows we adjust the three free effective parameters α, A and B of the lepton sector of our model to reproduce the experimental values of the five physical observables in the neutrino sector, i.e., three leptonic mixing parameters and two neutrino mass squared splittings, reported in Tables 1, 2, for the normal (NH) and inverted (IH) hierarchies of the neutrino mass spectrum, respectively. We fit only α to adjust the values of the leptonic mixing parameters $\sin^2\theta_{ij}$, whereas A and B for the normal (NH) and inverted (IH) mass hierarchies are given by:

$$\text{NH} : m_{\nu_1} = 0, \quad m_{\nu_2} = B = \sqrt{\Delta m_{21}^2} \approx 9\text{meV}, \quad m_{\nu_3} = 2A = \sqrt{\Delta m_{31}^2} \approx 51\text{meV}; \tag{39}$$

$$\text{IH}: m_{\nu_2} = B = \sqrt{\Delta m_{21}^2 + \Delta m_{13}^2} \approx 50\text{meV}, \;\; m_{\nu_1} = 2A = \sqrt{\Delta m_{13}^2} \approx 49\text{meV}, \;\; m_{\nu_3} = 0, \qquad (40)$$

as resulting from Eqs. (35), (34) and the definition $\Delta m_{ij}^2 = m_i^2 - m_j^2$. The best fit values of Δm_{ij}^2 have been taken from Tables 1 and 2 for the normal and inverted mass hierarchies, respectively.

We vary the model parameter α in Eq. (37) to fit the leptonic mixing parameters $\sin^2\theta_{ij}$ to the experimental values reported in Tables 1, 2. We obtain the following best fit result:

$$\text{NH} \;:\; \alpha = -0.88\pi, \quad \sin^2\theta_{12} \approx 0.34, \quad \sin^2\theta_{23} \approx 0.61, \quad \sin^2\theta_{13} \approx 0.0232; \qquad (41)$$

$$\text{IH} \;:\; \alpha = 0.12\,\pi, \quad \sin^2\theta_{12} \approx 0.34, \quad \sin^2\theta_{23} \approx 0.61, \quad \sin^2\theta_{13} \approx 0.0238. \qquad (42)$$

From the comparison of Eqs. (42), (41) with Tables 1, 2, it follows that $\sin^2\theta_{13}$ and $\sin^2\theta_{23}$ are in excellent agreement with the experimental data, for both normal and inverted mass hierarchies, whereas $\sin^2\theta_{12}$ is deviated 2σ away from its best fit values. This shows that the physical observables in the lepton sector obtained in our model are consistent with the experimental data. Furthermore, as previously mentioned, our model predicts a vanishing leptonic Dirac CP violating phase.

Parameter	$\Delta m_{21}^2(10^{-5}\text{eV}^2)$	$\Delta m_{31}^2(10^{-3}\text{eV}^2)$	$\left(\sin^2\theta_{12}\right)_{\text{exp}}$	$\left(\sin^2\theta_{23}\right)_{\text{exp}}$	$\left(\sin^2\theta_{13}\right)_{\text{exp}}$
Best fit	7.60	2.48	0.323	0.567	0.0234
1σ range	7.42 – 7.79	2.41 – 2.53	0.307 – 0.339	0.439 – 0.599	0.0214 – 0.0254
2σ range	7.26 – 7.99	2.35 – 2.59	0.292 – 0.357	0.413 – 0.623	0.0195 – 0.0274
3σ range	7.11 – 8.11	2.30 – 2.65	0.278 – 0.375	0.392 – 0.643	0.0183 – 0.0297

Table 1: Range for experimental values of neutrino mass squared splittings and leptonic mixing parameters, taken from Ref. [35], for the case of normal hierarchy.

Parameter	$\Delta m_{21}^2(10^{-5}\text{eV}^2)$	$\Delta m_{13}^2(10^{-3}\text{eV}^2)$	$\left(\sin^2\theta_{12}\right)_{\text{exp}}$	$\left(\sin^2\theta_{23}\right)_{\text{exp}}$	$\left(\sin^2\theta_{13}\right)_{\text{exp}}$
Best fit	7.60	2.38	0.323	0.573	0.0240
1σ range	7.42 – 7.79	2.32 – 2.43	0.307 – 0.339	0.530 – 0.598	0.0221 – 0.0259
2σ range	7.26 – 7.99	2.26 – 2.48	0.292 – 0.357	0.432 – 0.621	0.0202 – 0.0278
3σ range	7.11 – 8.11	2.20 – 2.54	0.278 – 0.375	0.403 – 0.640	0.0183 – 0.0297

Table 2: Range for experimental values of neutrino mass squared splittings and leptonic mixing parameters, taken from Ref. [35], for the case of inverted hierarchy.

In the following we proceed to determine the effective Majorana neutrino mass parameter, which is proportional to the amplitude of neutrinoless double beta ($0\nu\beta\beta$) decay. This effective Majorana neutrino mass parameter has the form:

$$m_{\beta\beta} = \left|\sum_j U_{ek}^2 m_{\nu_k}\right|, \qquad (43)$$

where U_{ej}^2 and m_{ν_k} are the PMNS mixing matrix elements and the Majorana neutrino masses, respectively.

Using Eqs. (36), (39), (40) and (43), it follows that the effective Majorana neutrino mass parameter, for both Normal and Inverted hierarchies, acquires the following values:

$$m_{\beta\beta} = \begin{cases} 2 \text{ meV} & \text{for} \quad \text{NH} \\ 47 \text{ meV} & \text{for} \quad \text{IH} \end{cases} \tag{44}$$

Our results for the effective Majorana neutrino mass parameter given above, are beyond the reach of the present and forthcoming $0\nu\beta\beta$ decay experiments. The EXO-200 experiment [26] sets the current best upper limit on the effective neutrino mass parameter equal to $m_{\beta\beta} \leq 160$ meV, correspoding to $T_{1/2}^{0\nu\beta\beta}(^{136}\text{Xe}) \geq 1.6 \times 10^{25}$ yr at 90% C.L. This bound will be improved within the not too distant future. The GERDA "phase-II"experiment [27, 28] is expected to reach $T_{1/2}^{0\nu\beta\beta}(^{76}\text{Ge}) \geq 2 \times 10^{26}$ yr, which corresponds to $m_{\beta\beta} \leq 100$ meV. A bolometric CUORE experiment, using ^{130}Te [29], is currently under construction. This experiment features an estimated sensitivity of about $T_{1/2}^{0\nu\beta\beta}(^{130}\text{Te}) \sim 10^{26}$ yr, corresponding to an effective Majorana neutrino mass parameter $m_{\beta\beta} \leq 50$ meV. Besides that, there are proposals for ton-scale next-to-next generation $0\nu\beta\beta$ experiments using ^{136}Xe [30, 33] and ^{76}Ge [27, 32], which claim sensitivities over $T_{1/2}^{0\nu\beta\beta} \sim 10^{27}$ yr, corresponding to an effective Majorana neutrino mass parameter $m_{\beta\beta} \sim 12 - 30$ meV. For a recent review, see for example Ref. [34]. Consequently, the Eq. (44) indicates that our model predicts $T_{1/2}^{0\nu\beta\beta}$ at the level of sensitivities of the next generation or next-to-next generation $0\nu\beta\beta$ experiments.

4. Quark masses and mixing.

From the quark Yukawa terms of Eq. (9) and the relation given by Eq. (16), we find that the SM quarks do not mix with the exotic quarks and that the SM quark mass matrices are:

$$M_U = \begin{pmatrix} a_1^{(U)}\lambda^8 & 0 & 0 \\ 0 & a_2^{(U)}\lambda^4 & 0 \\ 0 & 0 & a_3^{(U)} \end{pmatrix} \frac{v}{\sqrt{2}}, \qquad M_D = \begin{pmatrix} a_{11}^{(D)}\lambda^7 & a_{12}^{(D)}\lambda^6 & a_{13}^{(D)}\lambda^6 e^{-i\delta_q} \\ a_{21}^{(D)}\lambda^6 & a_{22}^{(D)}\lambda^5 & a_{23}^{(D)}\lambda^5 \\ a_{31}^{(D)}\lambda^6 e^{-i\delta_q} & a_{32}^{(D)}\lambda^5 & a_{33}^{(D)}\lambda^3 \end{pmatrix} \frac{v}{\sqrt{2}}, \tag{45}$$

where $\lambda = 0.225$ is one of the Wolfenstein parameters, $v = 246$ GeV the scale of electroweak symmetry breaking and $a_{ij}^{(U,D)}$ $(i, j = 1, 2, 3)$ are $O(1)$ parameters.
Moreover, we find that the exotic quark masses are:

$$m_T = y^{(T)}\frac{v_\chi}{\sqrt{2}}, \qquad m_{J^1} = y_1^{(J)}\frac{v_\chi}{\sqrt{2}} = \frac{y_1^{(J)}}{y^{(T)}} m_T, \qquad m_{J^2} = y_2^{(J)}\frac{v_\chi}{\sqrt{2}} = \frac{y_2^{(J)}}{y^{(T)}} m_T. \tag{46}$$

Since the charged fermion mass and quark mixing pattern emerges from the breaking of the $Z_4 \otimes Z_6 \otimes Z_{16}$ discrete group and in order to simplify the analysis, the following scenario is considered:

$$\arg\left(a_{13}^{(D)}\right) = \arg\left(a_{31}^{(D)}\right), \qquad a_{ij}^{(D)} = a_{ji}^{(D)}, \qquad i, j = 1, 2, 3. \tag{47}$$

Besides that, to show that the quark textures given above can fit the experimental data, and in order to simplify the analysis, we adopt a benchmark where we set $a_1^{(U)} = a_3^{(U)} = 1$ and $a_{22}^{(U)} = a_{33}^{(D)}$, as suggested by naturalness arguments and by the relation $m_c \sim m_b$, respectively. Then, we proceed to fit the parameters $a_{11}^{(D)}$ $a_{22}^{(D)}$, $a_{33}^{(D)}$, $a_{12}^{(D)}$, $a_{13}^{(D)}$, $a_{23}^{(D)}$ and the phase δ_q, to reproduce the 10

physical observables of the quark sector, i.e., the six quark masses, the three mixing angles and the CP violating phase. The obtained values for the quark masses, the three quark mixing angles and the CP violating phase δ in Table 3 correspond to the best fit values:

$$\begin{aligned} a_{11}^{(D)} &\simeq 1.11, \qquad a_{22}^{(D)} \simeq 0.59, \qquad a_{12}^{(D)} \simeq 0.54, \\ a_{13}^{(D)} &\simeq 0.43, \qquad a_{23}^{(D)} \simeq 1.13, \qquad a_{33}^{(D)} \simeq 1.42, \qquad \delta_q \simeq 66^\circ. \end{aligned} \tag{48}$$

Observable	Model value	Experimental value
$m_u(MeV)$	1.14	$1.45^{+0.56}_{-0.45}$
$m_c(MeV)$	635	635 ± 86
$m_t(GeV)$	173.9	$172.1 \pm 0.6 \pm 0.9$
$m_d(MeV)$	2.9	$2.9^{+0.5}_{-0.4}$
$m_s(MeV)$	57.7	$57.7^{+16.8}_{-15.7}$
$m_b(GeV)$	2.82	$2.82^{+0.09}_{-0.04}$
$\sin\theta_{12}$	0.225	0.225
$\sin\theta_{23}$	0.0412	0.0412
$\sin\theta_{13}$	0.00352	0.00351
δ	66°	68°

Table 3: Model and experimental values of the quark masses and CKM parameters.

The obtained quark masses, quark mixing angles and CP violating phase exhibit an excellent agreement with the experimental data. Let us note, that despite the aforementioned simplifying assumptions that allow us to eliminate some of the free parameters, a good fit with the low energy quark flavor data is obtained, showing that our model is indeed capable of a very good fit to the experimental data of the physical observables for the quark sector. The obtained and experimental values for the physical observables of the quark sector are reported in Table 3. We use the experimental values of the quark masses at the M_Z scale, from Ref. [53] (which are similar to those in [54]), whereas the experimental values of the CKM parameters are taken from Ref. [6].

In what follows we briefly comment about the phenomenological implications of our model in the flavor changing processes involving quarks. As previously mentioned, the different Z_3 charge assignments for SM and exotic right handed quark fields imply the absence of mixing between them. Due to the absence of mixings between SM and exotic quarks, the exotic T, J^1 and J^2 quarks do not exhibit flavor changing neutral decays into SM quarks and gauge bosons, SM light 126 GeV Higgs boson and SM quarks. Thus, assuming that the H_2^0 and $\overline{H}_2^0$ neutral Higgs bosons are heavier than the exotic T, J^1 and J^2 quarks, it follows that the flavor changing neutral exotic quark decays are absent in our model. Consequently these exotic quarks can be searched at the LHC via their flavor changing charged decays into SM quarks and gauge bosons, specifically in their dominant decay modes $T \to bW$ and $J^{1,2} \to tW$. These exotic quarks can be produced at the LHC via Drell-Yan proccesses mediated by charged gauge bosons, where the final states will include the exotic T quark with a SM down type quark as well as any of the exotic J^1 or J^2 quarks with a SM up type quark. Furthermore, from the quark Yukawa terms, one can easily see

that the our model predicts the absence of flavor changing top quark decays $t \to hc$ and $t \to hu$ at tree level. The flavor changing top quark decays $t \to hc$ and $t \to hu$ only arise at one loop level and will involve virtual charged gauge bosons and exotic quarks running in the loops. Thus, a measurement of the branching fraction for the $t \to hc$ and $t \to hu$ decays at the LHC will be crucial for confirming or ruling out our model. It would be interesting to perform a detailed study of the exotic quark production at the LHC, the exotic quark decay modes and the flavor changing top quark decays. This is beyond the scope of this work and is left for future studies.

5. Conclusions

We constructed a predictive $SU(3)_C \otimes SU(3)_L \otimes U(1)_X$ model with $\beta = -\frac{1}{\sqrt{3}}$, based on the A_4 flavor symmetry supplemented by the $Z_3 \otimes Z_4 \otimes Z_6 \otimes Z_{16}$ discrete group. Our model successfully accounts for the observed fermion masses and mixing angles. The obtained values for the physical observables in both quark and lepton sectors exhibit an excellent agreement with the experimental data. The A_4, Z_4 and Z_3 symmetries allow to reduce the number of parameters in the Yukawa terms, increasing the predictivity power of the model. The breaking of the $Z_4 \otimes Z_6 \otimes Z_{16}$ discrete group at high energy, gives rise to the observed charged fermion mass pattern and quark mixing hierarchy. In our model the Majorana neutrinos acquire very small masses, much smaller than the Dirac neutrino masses, thus giving rise to an inverse seesaw mechanism for the generation of the light active neutrino masses. In this scenario, the spectrum of neutrinos includes very light active neutrinos and TeV scale pseudo Dirac nearly degenerate sterile neutrinos. Our model predicts a vanishing leptonic Dirac CP violating phase as well as an effective Majorana neutrino mass, relevant for neutrinoless double beta decay, with values $m_{\beta\beta} = 2$ and 48 meV, for the normal and the inverted hierarchies, respectively. For the inverted hierarchy neutrino mass spectrum, our obtained value of 48 meV for the effective Majorana neutrino mass is within the declared reach of the next generation bolometric CUORE experiment [29] or, more realistically, of the next-to-next generation tone-scale $0\nu\beta\beta$-decay experiments. Under the assumption that the exotic T, J^1 and J^2 quarks are lighter than the H_2^0 and $\overline{H}_2^0$ neutral Higgs bosons, our model predicts the absence of the flavor changing neutral exotic quark decays, which implies that they can be searched at the LHC via their dominant flavor changing charged decay modes $T \to bW$ and $J^{1,2} \to tW$. Furthermore, our model predicts the absence of flavor changing neutral top quark decays at tree level, implying that they occur at one loop level. Possible directions for future work along these lines would be to study the constraints on the heavy charged gauge boson masses in our model arising from the upper bound on the branching fraction for the flavor changing top quark decays, the oblique parameters, the $Zb\overline{b}$ vertex and the Higgs diphoton signal strength. The heavy exotic quark decays and their production at the LHC may be useful to study. All these studies require carefull investigations that we left outside the scope of this work.

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Appendix A. The product rules for A_4

The A_4 group has one three-dimensional **3** and three distinct one-dimensional **1**, **1′** and **1″** irreducible representations, satisfying the following product rules:

$$\mathbf{3}\otimes\mathbf{3} = \mathbf{3}_s \oplus \mathbf{3}_a \oplus \mathbf{1} \oplus \mathbf{1}' \oplus \mathbf{1}'', \tag{A.1}$$

$$\mathbf{1}\otimes\mathbf{1} = \mathbf{1}, \qquad \mathbf{1}'\otimes\mathbf{1}'' = \mathbf{1}, \qquad \mathbf{1}'\otimes\mathbf{1}' = \mathbf{1}'', \qquad \mathbf{1}''\otimes\mathbf{1}'' = \mathbf{1}',$$

Considering (x_1, y_1, z_1) and (x_2, y_2, z_2) as the basis vectors for two A_4-triplets **3**, the following relations are fullfilled:

$$\begin{aligned}
(\mathbf{3}\otimes\mathbf{3})_{\mathbf{1}} &= x_1y_1 + x_2y_2 + x_3y_3, \\
(\mathbf{3}\otimes\mathbf{3})_{\mathbf{3}_s} &= (x_2y_3 + x_3y_2, x_3y_1 + x_1y_3, x_1y_2 + x_2y_1), \quad (\mathbf{3}\otimes\mathbf{3})_{\mathbf{1}'} = x_1y_1 + \omega x_2y_2 + \omega^2 x_3y_3, \\
(\mathbf{3}\otimes\mathbf{3})_{\mathbf{3}_a} &= (x_2y_3 - x_3y_2, x_3y_1 - x_1y_3, x_1y_2 - x_2y_1), \quad (\mathbf{3}\otimes\mathbf{3})_{\mathbf{1}''} = x_1y_1 + \omega^2 x_2y_2 + \omega x_3y_3,
\end{aligned} \tag{A.2}$$

where $\omega = e^{i\frac{2\pi}{3}}$. The representation **1** is trivial, while the non-trivial **1′** and **1″** are complex conjugate to each other. Some reviews of discrete symmetries in particle physics are found in Refs. [8, 9, 10, 55].

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